

$$c_{11}^D = c_{11}^E \left( 1 + \frac{e_{11}^2}{\epsilon_{11} c_{11}^E} \right),$$

$$c_{111}^D = c_{111}^E \left[ 1 + \frac{3e_{11}}{\epsilon_{11} c_{111}^E} \left( \frac{\partial e_{11}}{\partial \eta_1} \right) - \frac{e_{11}^2}{\epsilon_{11} c_{111}^E} \left( \frac{1}{\epsilon_{11}} \frac{\partial \epsilon_{11}}{\partial \eta_1} \right) \right],$$

and

$$c_{1111}^D = c_{1111}^E \left[ 1 + \frac{1}{\epsilon_{11} c_{1111}^E} \left( \frac{\partial e_{11}}{\partial \eta_1} \right)^2 - \frac{3e_{11}}{\epsilon_{11} c_{1111}^E} \left( \frac{1}{\epsilon_{11}} \frac{\partial \epsilon_{11}}{\partial \eta_1} \right) \left( \frac{\partial e_{11}}{\partial \eta_1} \right) + \frac{e_{11}^2}{\epsilon_{11} c_{1111}^E} \left( \frac{1}{\epsilon_{11}} \frac{\partial \epsilon_{11}}{\partial \eta_1} \right)^2 \right].$$

Substituting values from the present paper into these expressions gives the result that

$$c_{11}^D = c_{11}^E (1 + 0.0084),$$

$$c_{111}^D = c_{111}^E (1 + 0.11 - 0.0011),$$

and

$$c_{1111}^D = c_{1111}^E (1 + 0.023 - 0.0021 + 0.00002).$$

The contribution of piezoelectric stiffening to the third-order constants is calculated to be 11%. This value is large enough such that it should be observable in ultrasonic experiments conducted on low resistivity and open-circuited samples. The contributions to stiffening due to the nonlinear converse-effect constant  $[(1/\epsilon)(\partial\epsilon/\partial\eta)]$  are noted to be very much less than the contribution of the direct-effect nonlinear constant  $\partial e/\partial\eta$ .

#### VII. SUMMARY

The present paper reports the first quantitative experimental measurement of a nonlinear piezoelectric constitutive relation. The measurements yield the most accurate value reported to date for the linear piezoelectric stress constant of X-cut quartz, the first quantitative measurement of a nonlinear piezoelectric constant for the direct effect, and a value for the strain dependence of the

permittivity. The longitudinal finite-strain elastic constitutive relation for x-axis compression was determined with second-, third-, and fourth-order elastic-constant measurements which represent the compression of the solid to compressive strains up to  $4.3 \times 10^{-2}$ . The third-order constant value appears to differ significantly from the value determined from ultrasonic experiments. The technique seems generally applicable to the study of other piezoelectric solids which exhibit large elastic limits under shock-wave loading and should permit nonlinear piezoelectric constants to be studied in some detail.

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<sup>1</sup>For definitions of crystallographic orientations and piezoelectric constants see Proc. IRE **37**, 1378 (1949).

<sup>2</sup>The direct piezoelectric effect causes a piezoelectric polarization upon application of a stress or strain. The converse piezoelectric effect, ordinarily utilized for measurements of piezoelectric constants, causes a strain upon application of an electric field. See, e.g., W. G. Cady [*Piezoelectricity* (Dover, New York, 1964), Vol. I]. The present experiments provide a measure of the strain dependence of a piezoelectric stress constant. This nonlinear constant will be referred to as the direct nonlinear piezoelectric stress constant to distinguish it from the converse nonlinear piezoelectric constant which describes the change of piezoelectric constant with electric field.

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